The entropy of the image histogram as a measure of the performance of the NDVI vegetation index-a probabilistic approach using ALOS digital images*

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ABSTRACT: In this paper a methodology based on probability theory is applied to study the entropy *H* of images produced by the NDVI vegetation index. Theoretical considerations show that *H* is relatively low when the standard deviation *stdev* of the NIR band is equal to that of the Red band and it increases when the two *stdev* values differ considerably each other. Experimentation with ALOS images from Japan and Greece shows that theoretical predictions agree in qualitative terms (behavior of *H*) with real data. A modified normalized differences vegetation index (MNDVI), which controls the entropy value, is applied on the ALOS image over Japan to show that certain targets of interest which are covered by shadows may be more clearly expressed. The conclusions of the present paper may be useful in mapping land cover with ALOS imagery.
Key-words: *NDVI, MNDVI, entropy, standard deviation, parameter c, semivariogram*.

ΠΕΡΙΛΗΨΗ: Σε αυτήν την εργασία εφαρμόζεται μια μεθοδολογία βασισμένη στη θεωρία πιθανοτήτων, προκειμένου να μελετηθεί η συμπεριφορά της εντροπίας *H* ψηφιακής εικόνας του δείκτη βλάστησης NDVI. Η θεωρητική προσέγγιση δείχνει ότι το *H* λαμβάνει σχετικά χαμηλές τιμές όταν η τυπική απόκλιση *stdev* της φασματικής ζώνης εγγύς υπερύθρου είναι ίση με αυτήν της ζώνης ερυθρού και αυξάνεται στο βαθμό που οι δυο τιμές τυπικής απόκλισης διαφέρουν σημαντικά μεταξύ τους. Ο πειραματισμός με δορυφορικές εικόνες ALOS από περιοχές της Ιαπωνίας και της Ελλάδας δείχνει ότι οι θεωρητικές προβλέψεις συμφωνούν, με ποιοτικούς όρους συμπεριφοράς των τιμών *H*, με τα πραγματικά δεδομένα. Στη δορυφορική εικόνα από περιοχή της Ιαπωνίας εφαρμόζεται ένας τροποποιημένος δείκτης βλάστησης MNDVI, μια χαρακτηριστική παράμετρος του οποίου διαμορφώνει την τιμή της εντροπίας. Διαπιστώνεται ότι αυτός ο τροποποιημένος δείκτης βλάστησης αναδεικνύει ευκρινώς περιοχές που καλύπτονται από σκιές και δεν φαίνονται καθαρά στην αρχική πολυφασματική εικόνα. Τα συμπεράσματα της παρούσας εργασίας μπορούν να αξιοποιηθούν στη χαρτογράφηση εδαφών με δορυφορικές εικόνες.
Λέξεις-κλειδιά: *NDVI, MNDVI, εντροπία, παράμετρος c, ημιβαριόγραμμα*.

INTRODUCTION

In geological and environmental research, spectral band ratios are often used as vegetation indices, for mapping the vegetation cover of the area under study. Various vegetation indices have been proposed, mainly based on empirical criteria of response over vegetation types, soils or geological targets of interest (JENSEN, 1996; FAUST, 1989; BURGAN, 1996; RONDEAUX *et al.*, 1996). A great effort has been done in extracting information about vegetation cover parameters such as the Leaf Area Index (LAI) or the forest biomass, using vegetation indices and various reflectance bands (GITELSON, 2004; MALLINIS *et al.*, 2004; KALE *et al.*, 2005; SILLEOS *et al.*, 2006). In these approaches, remote sensing data are compared with ground data and correlation coefficients are computed. In certain cases, the sensitivity of various vegetation indices is assessed with the aid of mathematical models which associate the vegetation cover with its reflectance at various bands (GOEL, 1988; VERHOEF, 1998; HABOUDANE *et al.*, 2004).

The authors of the present paper have developed an alternative methodology of assessing the efficiency of vegetation indices with the aid of probability theory, which has already been applied to the study of the NDVI and the Simple Vegetation Index SVI (VAIOPOULOS *et al.*, 2004) the Soil Adjusted Vegetation Index SAVI (SKIANIS *et al.*, 2004) and the Transformed Vegetation Index TVI (SKIANIS *et al.*, 2007). The methodological difference between this approach and other efforts is that our research is centred on the statistical behaviour of the vegetation index under study, using theorems about bivariate distributions and appropriate density functions to describe the histograms of the frequency bands. This means that the whole approach is focused on the mathematical structure of the function which describes the vegetation index and not on its functionality at different ground types or its correlation with biophysical parameters.

The basic idea of the probabilistic approach, which has been elaborated so far, is that an image of a vegetation index

* Η εντροπία του ιστογράμματος εικόνας ως μέτρο της λειτουργικότητας του δείκτη βλάστησης NDVI-μια πιθανοθεωρητική προσέγγιση με αξιοποίηση εικόνων ALOS

with a large standard deviation has a good contrast, which may help in detecting targets with a different tonality. This paper is centred on how the entropy of the extensively used NDVI vegetation index behaves compared to its standard deviation. Since the entropy of an image is a measure of its information content, it is reasonable to assume that its value is also relevant to the clarity of the targets of interest and is correlated with the standard deviation of the histogram. The theoretical predictions are compared with real data obtained by ALOS images over Japan and Greece. Finally, the performance of a modified NDVI vegetation index is studied.

THEORETICAL CONSIDERATIONS

According to the probabilistic approach, it is assumed that the histogram of a band has a zero frequency for a null brightness value, presents a peak at a relatively low value of the tonality range and gets practically nullified at high brightness values (SCHOWENGERDT, 1997). A satellite image, from which the atmospheric scattering has been removed, is expected to have such a behaviour, if histogram stretch has not been done. In practice, the behaviour of the histogram may be more complicated and it may present more than one peaks, but it is reasonable to make a rough approximation of the histogram by a simple distribution, which may help in the mathematical analysis.

According to VAIOPOULOS *et. al* (2004), the distribution $p(x)$ which describes the histogram of a spectral band is given by:

$$p(x) = 2ax.e^{-ax^2} \qquad (1)$$

x is the reflectance of a pixel at the Red or NIR band. Parameter a controls the shape of the curve $p(x)$ and it is inversely proportional to the square of the standard deviation of $p(x)$.

In Fig. 1 the distribution $p(x)$ for various a values is presented. As long as a increases the width of the distribution decreases.

The NDVI vegetation index is defined by (ROUSE *et al.*, 1973):

$$u = \frac{NIR - \mathrm{Re}\,d}{NIR + \mathrm{Re}\,d} \qquad (2)$$

u is the value of the vegetation index. NIR and $\mathrm{Re}d$ are the ground reflectances at these two spectral bands.

Assuming that the NIR and the Red bands are independent each other and using the distribution $p(x)$ of Eq. 1 to describe the histograms of the two bands, it can be proved, with the aid of probability theory, that the distribution $g(u)$ of the NDVI is given by (VAIOPOULOS *et. al*, 2004):

$$g(u) = \frac{4\lambda(1-u^2)}{\left[\lambda(1+u)^2 + (1-u)^2\right]^2} \qquad (3)$$

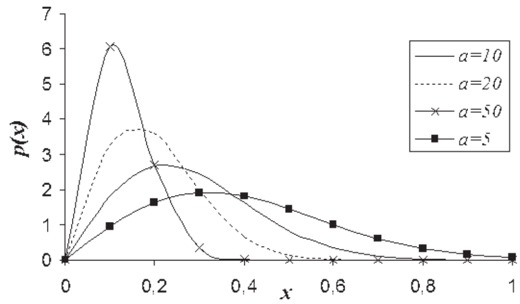

Fig. 1. A graphical representation of distribution *p(x)* for a = 10.

$$\lambda = \frac{[stdev(\mathrm{Re}\,d)]^2}{[stdev(NIR)]^2} = \frac{a_1}{a_2} \qquad (4)$$

u takes values from −1 to +1.

Parameter λ is defined by (VAIOPOULOS *et al.*, 2004):

stdev( ) is the standard deviation of the spectral band inside the parenthesis. a_1 and a_2 are the values of the parameter a of distribution p for the NIR and Red zone, respectively.

In Fig. 2 the NDVI distribution $g(u)$ is presented, for various λ values. It can be observed that for λ more than unity the peak of the distribution is shifted to the left part of the u axis (low u values). This means that for $\lambda > 1$ the NDVI image is expected to be dominated by dark tones. On the other hand, for λ less than unity, the peak of $g(u)$ is shifted to the right and the image is expected to have a bright tonality. For λ equal to unity the distribution has its peak at $u = 0$.

The entropy H of $g(u)$ is defined by:

$$H = \int_{-1}^{1} g(u) \cdot \ln[g(u)]du \qquad (5)$$

The definition of H is a modification of the expression which appears in WIENER (1961), where $\log_2$ instead of the natural logarithm ln is used and a negative sign at the left of the integral is included. $\log_2$ is preferable when the pixel tonality is directly expressed as an integer between 0 and 2^n-1 (n is an integer). In the case of a vegetation index, u values

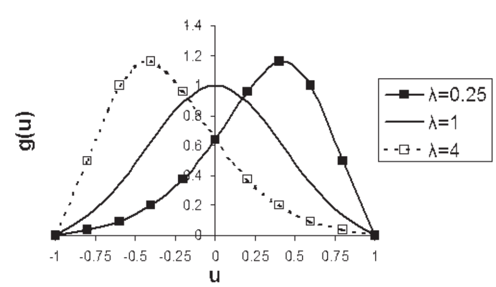

Fig. 2. The NDVI distribution *g(u)* for various λ values.

are expressed as real numbers, therefore it is better to use the natural logarithm. The minus sign is used in the theory of image compression, where positive entropy values are preferred to define the number of binary digits which should be kept to express the brightness of a pixel in the compressed image. In the present paper, the entropy is treated as a measure of the quantity of information of a digital image, or, in other words, of the width and shape of the image histogram, therefore the minus sign is not needed.

The mean value μ of the distribution $g(u)$ is given by:
And the standard deviation *stdev* is:

$$\mu = \int_{-1}^{1} u \cdot g(u)du \qquad (6)$$

$$stdev = \sqrt{\int_{-1}^{1} (u-\mu)^2 g(u)du} \qquad (7)$$

The integrals of the Eqs. 5, 6 and 7 may be numerically calculated, putting $g(u)$ equal to the right part of Eq. 3, for various values of the parameter λ.

In Fig. 3 the entropy H of the distribution $g(u)$ of the NDVI vegetation index against λ is presented. It can be observed that H is minimum at λ equal to unity.

In Fig. 4 the standard deviation *stdev* of $g(u)$ against λ is presented. It can be observed that *stdev* takes its maximum value for λ equal to unity.

The standard deviation does not change much with the parameter λ. On the other hand, the variation of H with λ is more pronounced and its behaviour is the opposite of that of *stdev*. In this paper, NIR and Red bands are assumed to be uncorrelated, in order to focus on the influence of λ on the behaviour of the NDVI histogram. However if the NIR and Red bands are correlated, then the entropy, as well as the *stdev*, tend to decrease as long as the correlation coefficient increases (Skianis *et al*., 2008). A comparison of Fig. 2 with Fig. 3 shows that the entropy H is strongly dependent on the shape of the histogram of the NDVI image and it takes a minimum value when the histogram is symmetric around its peak.

The theoretical predictions about the behaviour of the entropy and the standard deviation of an NDVI image have to be tested with real data obtained by satellite imagery.

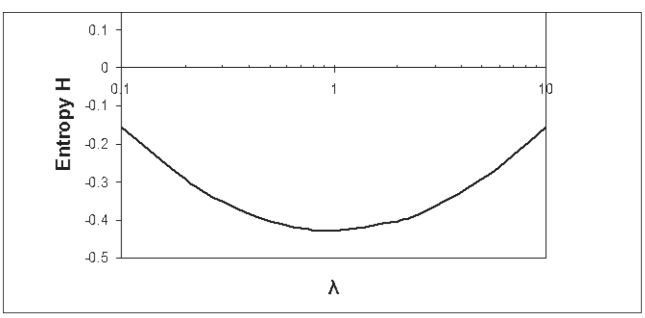

Fig. 3. Entropy H of the NDVI distribution *g(u)* against λ.

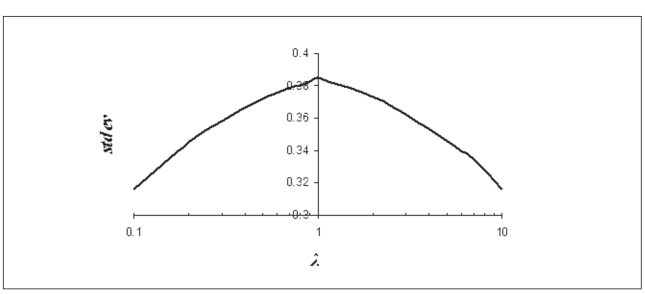

Fig. 4. Standard deviation *stdev* of *g(u)* against λ.

EXPERIMENTATION WITH ALOS IMAGES

In order to compare the theoretical predictions with actual data, four ALOS images from Greece and Japan were processed. In Fig. 5 the satellite image over a region of Japan is presented. This picture represents the NIR channel of the satellite image. At the eastern and north eastern part, some clouds and their shadows prevent the land cover to be expressed. One of the shadowed regions, with a dark tonality, is marked by an ellipse. The vegetation cover is expressed with brighter tones. In Fig. 6, the NIR band of an ALOS image over the area of Attica, Greece is presented. The arrow points at a mountainous region with dark tones, which was burnt shortly before the image was taken (July 2007). Two other images over the same area, one year before and one year after the fire, were also used in the context of the experimentation with ALOS images.

In Fig. 7 the NDVI image of Fig. 5 (ALOS image over Japan) is presented. It can be observed that the shadowed places of Fig. 5 are more clearly expressed.

In order to testify the theoretically predicted behaviour of the histogram of the NDVI vegetation index, the four ALOS images were separated in subsets (areas of interest). For each subset the quantities *stdev*(Re*d*), *stdev*(*NIR*) and λ were calculated, using the ERDAS Imagine software. Then, the

Fig. 5. An ALOS image, NIR channel, over a region of Japan. The ellipse surrounds shadowed formations.

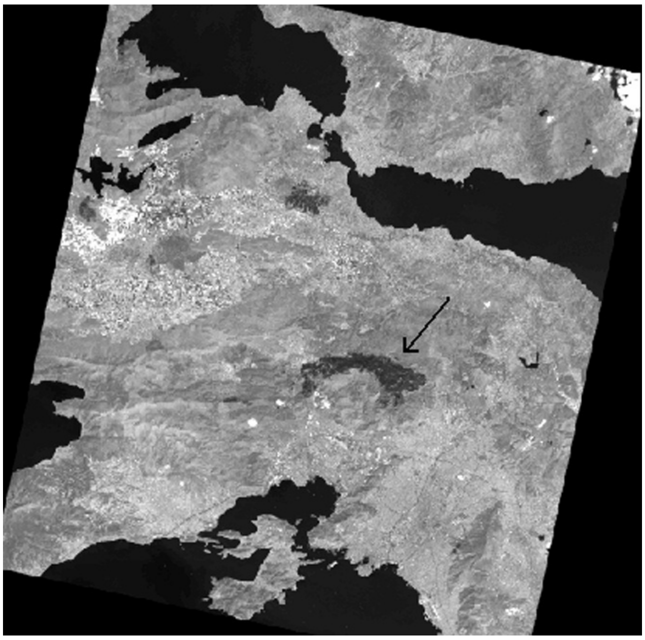

Fig. 6. An ALOS image NIR channel, over the region of Attica, Greece. The arrow points at a burnt area.

Fig. 7. The NDVI of the ALOS image over Japan.

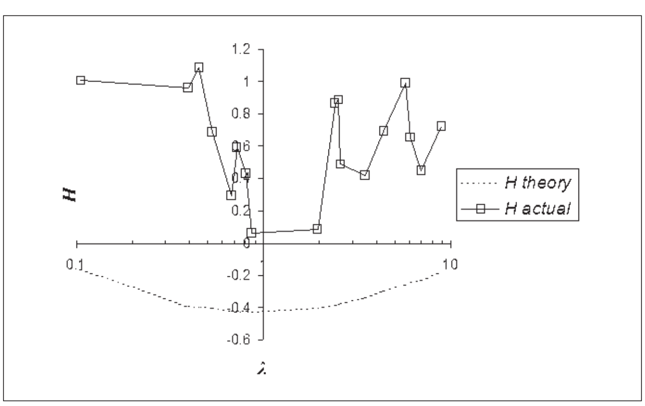

Fig. 8. A graphical representation of the theoretically predicted and actual entropy values *H* of the ALOS images.

NDVI image of each subset was produced and the standard deviation *stdev* of each image was calculated. Finally, the entropy *H* of each NDVI image was calculated, by exporting the histogram data to an Excel file and making the proper numerical integration. The theoretical *stdev* and *H* values for each λ were also calculated, according to the Eqs. 5 and 6. The theoretically predicted and actual (image) entropy values are presented in Fig. 8.

It can be observed that the actual entropy values present strong fluctuations and, in quantitative terms, they differ considerably from the theoretically predicted ones. In qualitative terms of behaviour however, there is a certain accordance between the theoretically predicted and actual variations of the entropy *H* with λ. Both curves present a minimum around λ equal to unity. As long as the behaviour of the standard deviation is concerned, it was also found that there is a qualitative accordance between the theoretically predicted variation of *stdev* with λ and the actual one (both curves present a maximum around λ equal to unity).

The general remark is that the probabilistic approach produces results, which, in qualitative terms, agree with real data. Two possible causes of the considerable quantitative deviations between theory and actual data, can be the complexity of the histograms of the NIR and Red bands (the distribution of the curve of Fig. 1 is a mere approximation of real histograms) and the significant correlation between NIR and Red reflectances, which have not been taken into account, because the mathematical treatment would be technically much more difficult and complicated. According to the authors opinion, the most important is that the probabilistic approach works and can help in assessing the functionality of the various proposed vegetation indices.

APPLICATION OF THE MODIFIED NORMALIZED DIFFERENCES VEGETATION INDEX (MNDVI)

In the ALOS image over a region of Japan, there are certain shadowed parts with dark tones (see Fig. 6). In the NDVI image of Fig. 7, these parts are more clearly expressed. This observation gives a stimulus to testify the performance of a Modified NDVI vegetation index, called MNDVI, which is defined by (Vaiopoulos *et al.*, 2004):

$$u = \frac{c \cdot NIR - \mathrm{Re}\,d}{c \cdot NIR + \mathrm{Re}\,d} \tag{8}$$

c is a real number, which generally takes values between 0.1 and 10. It can be proved (Vaiopoulos *et al.*, 2004) that the distribution $g(u)$ of the MNDVI is given by:

$$g(u) = \frac{4\lambda c^2(1-u^2)}{[\lambda(1+u)^2 + c^2(1-u)^2]^2} \tag{9}$$

Or, in a different way:

$$g(u) = \frac{(4\lambda / c^2)(1-u^2)}{[(\lambda / c^2)(1+u)^2 + (1-u)^2]^2} \qquad (10)$$

Comparing Eqs. (3) and (10), it can be realized that the MNDVI behaves like the NDVI with a parameter λ' equal to λ/c^2. Therefore the parameter c controls the statistical behaviour of the MNDVI in a similar way by which λ controls the NDVI. This means that different c values are expected to produce MNDVI images with different statistical parameters and optical effects. The maximum standard deviation and the minimum entropy are at $\lambda' = 1$, or $c = \sqrt{\lambda}$.

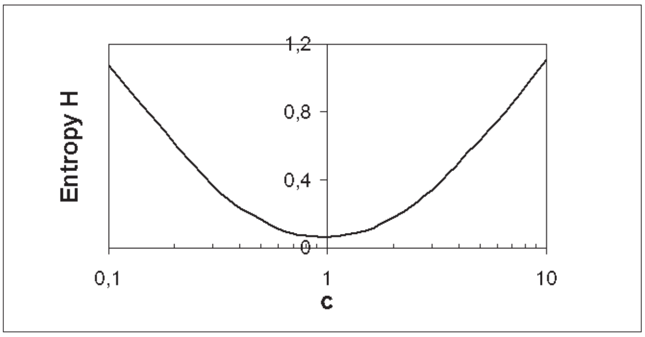

Fig. 9. The entropy H of the MNDVI images (region of Japan) against the parameter c.

The MNDVI was applied on the subset of the ALOS image over Japan, around the mouth of the river (see image of Fig. 5), for various c values. The standard deviation and entropy of each MNDVI image were calculated. Fig. 9 shows the variation of the entropy against c.

It can be observed that the minimum H is located at c approximately equal to unity, since the λ value of the ALOS subset image is equal to 0.862; therefore the c value for the peak of the H curve is $\sqrt{0.862}$ or 0.928, which is quite close to unity. It was also found that the maximum *stdev* value is also located at c approximately equal to unity.

The entropy and standard deviation of the MNDVI image histogram provide only certain information about the overall performance of this vegetation index, without taking into account its spatial structure. In order to see the spatial variation of the MNDVI values, in other words the variability of the tonality between pixels located at various distances each other, the semivariogram function $\gamma(h)$ of each MNDVI image was computed using the ILWIS software, according to the relation (Liang, 2004):

$$\gamma(h) = \frac{1}{2N}\sum_{i=1}^{N}\left[u(r_i) - u(r_i + h)\right]^2 \qquad (11)$$

$u(r_i)$ and $u(r_i + h)$ are the MNDVI values of a pair of pixels at locations r_i and $r_i + h$, which means that the pixels are separated by a lag h. N is the number of pairs of pixels separated by distance h.

In Fig. 10 the semivariograms of three MNDVI images, for $c = 0.3$, $c = 1$ and $c = 7$ are presented. It is obvious that the $c = 1$ semivariogram presents the steepest variation with distance h and has the highest sill (steady maximum value at large distances). Taking into account that for $c = 1$ the histogram standard deviation *stdev* is maximum and entropy H is minimum, leads to the conclusion that there is a certain correlation between these two statistical parameters of the whole MNDVI image and its semivariogram. An MNDVI image with a high histogram *stdev* and low H is expected to be an image with a strong spatial variation, which is expressed by a steep semivariogram with a high sill. If this is so, the probabilistic approach on the performance of the NDVI and MNDVI may provide an insight to the spatial variation of these vegetation indices, although it works with statistical parameters of the whole image histogram.

A high c value produces an MNDVI image with an optical effect which is quite different from that of a low c. This can be seen in Fig. 11, Fig. 12 and Fig. 13, which present the MNDVI of the ALOS image of Fig. 6, for c equal to 0.3, 1 and 7, respectively. The images of Fig. 11 and Fig. 12 are much the same, although Fig. 11 has a slightly darker tonality. The image of Fig. 13 is much brighter than the two others. Because of this, certain formations with a relatively high brightness for low c can not be clearly seen, because of saturation effects. On the other hand, the shadowed targets at the northern part appear more clearly, with a brighter and more variable tonality than in Fig. 11 and Fig. 12. In other words, the high c value has helped in expressing low tonality targets with an improved clarity and high tonality targets with a reduced clarity. The MNDVI was also applied to the three satellite images over Attica and it was found that the parameter c had a similar optical effect on the MNDVI images.

The general conclusion is that a low/high c produces an MNDVI image with an overall dark/bright tonality and a high entropy. An intermediate c value produces MNDVI images with an intermediate tonality and a low entropy. According to the targets of interest which have to be detected, one can use the MNDVI vegetation index with the suitable c. If for ex-

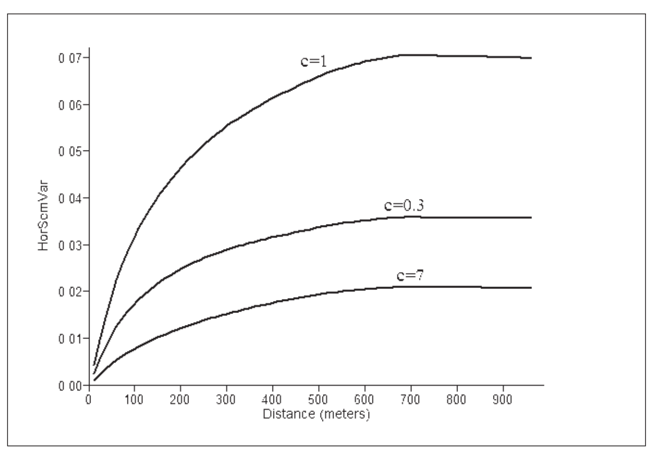

Fig. 10. The semivariograms of three MNDVI ALOS images over the same region of Japan, for c taking values 0.3, 1 and 7.

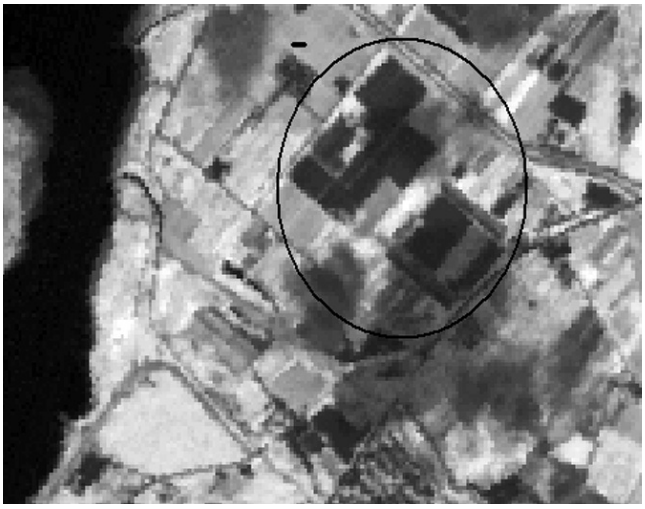

Fig. 11. An MNDVI image (region of Japan) with c = 0.3.

Fig. 12. An MNDVI image (region of Japan) with c = 1.

Fig. 13. An MNDVI image (region of Japan) with c = 7.

ample the target of interest is expressed with dark tones, as it happens in the case of a burnt area, a value of c between 5 and 10 may help in enhancing small tonality differences between adjacent parts of the target. These tonality differences may express variations on the intensity of the fire and its destructive consequences. Small values of c (less than unity) may be introduced in the expression for the MNDVI when areas with dense vegetation and high tonalities are supposed to be mapped and saturation effects in the image have to be avoided. The exact value of the proper c can be chosen empirically, trying on different values.

CONCLUSIONS

According to the mathematical analysis and the experimentation with the ALOS satellite image, the following conclusions may be drawn:

The entropy of the NDVI image depends on the standard deviation of the Red and the NIR channels. The entropy H is low for *stdev*(Red) approximately equal to *stdev*(NIR); and it increases for *stdev*(Red) considerably less or more than *stdev*(NIR). The behaviour of the standard deviation of the NDVI image is exactly the opposite: high values for *stdev*(Red) equal to *stdev*(NIR) and low values for *stdev*(Red) considerably less or more than *stdev*(NIR).

The MNDVI image may have a variable statistical behaviour with different optical effects, according to the value of the parameter c. If the c value is such that the MNDVI image has a low histogram entropy and a high standard deviation, then the spatial variation of the MNDVI is expected to be strong and its semivariogram will be steep and reach a high sill. The potential interpreter of the ALOS image is encouraged to try on different values of c, in order to detect targets of interest with different tonality ranges.

The results and conclusions of this paper may be useful in mapping the vegetation cover, recognizing different soil types, detecting burnt areas and removing shadows from a satellite image, which may be produced by topographic effects or cloud cover.

REFERENCES

BURGAN, R.E. (1996). Use of Remotely Sensed Data for Fire Danger Estimation. *Earsel Advances in Remote Sensing. Remote Sensing and GIS applications for Forest Fire Management*, 4 (4), 1-8

FAUST, N.L. (1989). Image Enhancement. *In* : KENT, A & J.G. WILLIAMS (*Eds*), *Encyclopedia of Computer Science and Technology*, Vol. 20, Supplement 5. Marcel Dekker Inc., New York, USA.

GITELSON, A.A. (2004). Wide Dynamic Range Vegetation Index for Remote Quantification of Biophysical Characteristics of Vegetation. *Journal of Plant Physiology* 161, 165-173 [doi: 10.1078/0176-1617-01176].

GOEL, N.S. (1988). Models of vegetation canopy reflectance and their use in estimation of biophysical parameters from reflectance data. *Remote Sens. Rev.*, 4, 1-212.

HABOUDANE, D., MILLER, J.R., PATTEY, E., ZARCO-TEJADA, P.J. & I.B. STRACHAN (2004). Hyperspectral vegetation indices and novel algorithms for predicting green LAI of crop canopies: Modeling and validation in the context of precision agriculture. *Remote Sens. Environ*. 90, 337-352 [doi: 10.1016/j.rse.2003.12.

013].

JENSEN, R.J. (1996). *Introductory Digital Image Processing: A Remote Sensing Perspective*. Prentice Hall, New Jersey, USA.

KALE, M., SINGH, S. & P.S. ROY (2005). Estimation of Leaf Area Index in dry deciduous forests from IRS-WiFS in Central India. *International Journal of Remote Sensing* 26(21), 4855-4867 [doi: 10.1080/01431160500181309].

LIANG, S. (2004). *Quantitative Remote Sensing of Land Surfaces*. Wiley, Hoboken, New Jersey, USA.

MALLINIS, G., KOUTSIAS, N., MAKRAS, A. & M. KARTERIS (2004). An assessment of the Information Content of Landsat –5 TM Bands for the estimation of forest stand parameters in Kassandra peninsula. *Proceedings of the 7th Panhellenic Geographical Congress*, Mytilini (Greece), October 14th-17th, Vvol. 2, 415-422.

RONDEAUX, G., STEVEN, M. & F. BARET (1996). Optimization of soil-adjusted vegetation indices. *Remote Sens. Environ*. 55, 95-107 [doi: 10.1016/0034-4257(95)00188-7].

ROUSE, J.W., HAAS, R.H., SCHELL, J.A. & D.W. DEERING (1973). Monitoring vegetation systems in the Great Plains with ERTS. *3rd ERTS Symposium*, vol. 1, 48-62.

SCHOWENGERDT, R.A. (1997). *Remote Sensing. Models and Methods for Image Processing*. Academic Press, San Diego, USA.

SILLEOS, N.G., ALEXANDRIDIS, T.K., GITAS, I.Z. & K. PERAKIS (2006). Advances made in Biomass Estimation and Vegetation Monitoring in the last 30 years. *Geocarto International* 21(4), 21-28 [doi: 10.1080/10106040608542399].

SKIANIS, G. AIM., NIKOLAKOPOULOS, K.G. & D.A. VAIOPOULOS (2008). The role of the correlation coefficient on the statistical behavior of a simple band ratio. *Proc. SPIE* 7109, 71090K (2008) [doi: 10.1117/12.799709].

SKIANIS, G. AIM., VAIOPOULOS, D. & K. NIKOLAKOPOULOS (2004). A study of the behavior of vegetation index SAVI, based on probability theory. *Proceedings of the 7th Panhellenic Congress of the Hellenic Geographical Society,* Mytilini (Greece), October 2004, vol. 1, 41-48.

SKIANIS, G. AIM., VAIOPOULOS, D. & K. NIKOLAKOPOULOS (2007). A Probabilistic Approach to the Problem of Assessing the Efficiency of the Transformed Vegetation Index. *Int. J. Sus. Dev. Plann*., 2(4), 461-480 [doi: 10.2495/SDP-V2_N4-461-480].

VAIOPOULOS, D., SKIANIS, G. AIM. & K. NIKOLAKOPOULOS (2004). The contribution of probability theory in assessing the efficiency of two frequently used vegetation indices. *International Journal of Remote Sensing* 25(20), 4219-4236.

VERHOEF, W. (1998). Theory of radiative transfer models applied in optical remote sensing of vegetation canopies. *Wageningen: Grafish Service Centrum Van Gils*.

WIENER, N. (1961). *Cybernetics*. Kastaniotis, Athens, Greece.