The Role of Ground Anisotropy in the Production of Self Potential Anomalies

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ABSTRACT: In the present paper we first present a review of the recent research on the behavior of the self potential field in a transversely anisotropic ground. In a transversely anisotropic medium the resistivity parallel to schistosity is different from that at perpendicular direction. It is pointed out that anisotropy may deform the characteristics of the self potential anomaly measured at ground surface. Such deformation may result important errors, if a quantitative interpretation is attempted without taking into account the ground anisotropy effect. It is possible, however, to make a reliable quantitative interpretation if the anisotropy parameters are known.

Then, we study the behavior of the self-potential field in the wave number domain. It is shown that transverse anisotropy functions as a filter which modulates the amplitude and phase of each component of the self-potential anomaly. The filter response is defined by the anisotropy parameters, mainly the schistosity angle (the angle between the plane of schistocity and the ground surface).

The results and conclusions of this paper may be useful in mineral exploration, as well as in detecting subsurface water flow and targets of geothermal activity.
Key-words: *anisotropy, schistosity, anisotropy coefficient, Fourier transform*.

ΠΕΡΙΛΗΨΗ: Στην παρούσα εργασία γίνεται πρώτα μια επισκόπηση της πρόσφατης ερευνητικής προσπάθειας που έχει πραγματοποιηθεί ως προς τη συμπεριφορά του πεδίου φυσικού δυναμικού σε εγκάρσια ανισότροπο έδαφος. Επισημαίνεται ότι η ανισοτροπία προξενεί στρεβλώσεις στη ροή του ηλεκτρικού ρεύματος και παραμορφώσεις στα χαρακτηριστικά της μετρούμενης ανωμαλίας φυσικού δυναμικού, με αποτέλεσμα να υπεισέρχονται σημαντικά σφάλματα κατά την ποσοτική ερμηνεία, αν θεωρηθεί ότι το υπέδαφος είναι ισότροπο. Είναι ωστόσο δυνατόν να γίνει μια αξιόπιστη ποσοτική ερμηνεία της ανωμαλίας φυσικού δυναμικού αν είναι γνωστές οι παράμετροι της ανισοτροπίας.

Στη συνέχεια μελετάται η συμπεριφορά του πεδίου φυσικού δυναμικού στο χώρο των κυματάριθμων και διαπιστώνεται ότι η εγκάρσια ανισοτροπία λειτουργεί ως φίλτρο που διαμορφώνει τα πλάτη και τις φάσεις των συνιστωσών της ανωμαλίας φυσικού δυναμικού. Η απόκριση του φίλτρου καθορίζεται από τις παραμέτρους της ανισοτροπίας και κυρίως από τη γωνία σχιστότητας.

Τα πορίσματα της παρούσας εργασίας μπορούν να αξιοποιηθούν στη γεωφυσική έρευνα για εντοπισμό μεταλλοφόρων σωμάτων, θερμοπηγών και υπόγειων ροών, όταν το υπέδαφος αποτελείται από σχιστολιθικά πετρώματα, τα οποία παρουσιάζουν μια ηλεκτρική ανισοτροπία.
Λέξεις-κλειδιά: *ανισοτροπία, σχιστότητα, συντελεστής ανισοτροπίας, μετασχηματισμός Fourier*.

INTRODUCTION

A rock or a geological formation exhibits transverse electrical anisotropy when the resistivity parallel to schistocity (or bedding) differs from that at perpendicular direction. Ground conductivity variations deform the electric current flow and consequently the measured at surface self potential (SP) anomaly. This deformation may produce errors when a quantitative interpretation of the SP anomaly is attempted.

Until middle of ’90s there was a limited number of references on the behavior of the electric field which is produced by a point pole in an anisotropic ground. These papers studied problems which appear when the mise-à-la-masse method is used in geophysical research (ASTEN, 1974; ELORANTA, 1988), or focused on the mathematical problem of a point pole in an anisotropic medium without making any comment about how anisotropy may influence the self potential field (LINDELL *et. al*., 1993; DAS, 1995; DAS & LI, 1996).

Recently, we have started researching on the behavior and the quantitative interpretation of the SP anomaly in a transversely anisotropic ground, which is produced by polarized bodies of various shapes (SKIANIS & HERNANDEZ, 1995, 1999; SKIANIS *et al*., 2000, 2001). This paper makes a review of the research on this subject and studies the behavior of the SP field in spatial frequency domain.

The results and conclusions of the present paper may be useful in mineral exploration and in detecting targets of geothermal activity and subsurface water flow.

* Ο ρόλος της ανισοτροπίας του υπεδάφους στην παραγωγή ανωμαλιών φυσικού δυναμικού.

THE MODEL OF THE POINT POLE

The model of the point pole with current intensity I at depth h, which is embedded in a transversely anisotropic ground with anisotropy coefficient A and schistosity angle θ, is the simplest one for studying the problem of anisotropy. This model may also help in elaborating other models with a more complex polarization geometry. The geometry of the problem is presented in (Fig. 1).

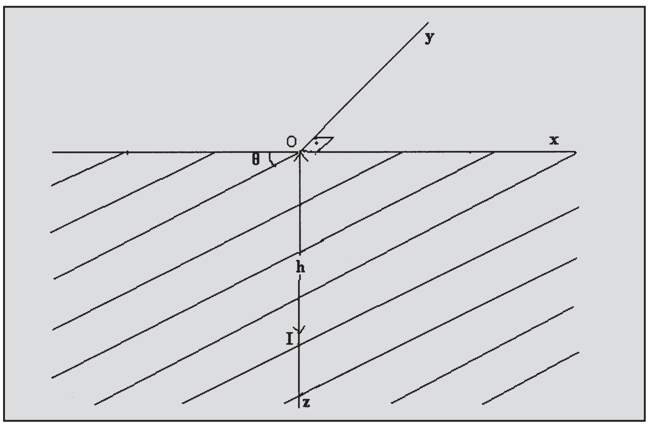

Fig. 1. The model of the point pole.

Using the image technique for an anisotropic half-space (LINDELL *et al.*, 1993), it can be proved (SKIANIS & HERNÀNDEZ, 1999) that self potential V(x, y) at ground surface is given by:

$$V(x,y)=\frac{I\rho_m}{2\pi\sqrt{\cos^2\theta+A^2\sin^2\theta}\cdot\sqrt{(x-x_0)^2+\dfrac{y^2}{\cos^2\theta+A^2\sin^2\theta}+h'^2}} \quad (1)$$

ϱ_m is the effective resistivity of the anisotropic ground and it is determined by the square root of the product of ground resistivity at schistosity level times the resistivity at the perpendicular direction.

The quantities x_0 and h are defined by:

$$x_0=\frac{(A^2-1)\sin\theta\cdot\cos\theta}{\cos^2\theta+A^2\sin^2\theta}\cdot h \quad (2)$$

$$h'=\frac{A}{\cos^2\theta+A^2\sin^2\theta}h \quad (3)$$

respectively.

The physical meaning of relation (1) is that in a homogeneous and transversely anisotropic ground, the SP anomaly which is produced by a point pole at depth h and horizontal location 0, behaves as if it were produced by a point pole embedded in a homogeneous and isotropic ground, at depth h' and horizontal location x_0.

Ground anisotropy may seriously deform the self potential anomaly and produce important errors in the calculation of the parameters of the point pole, if anisotropy is not taken into account.

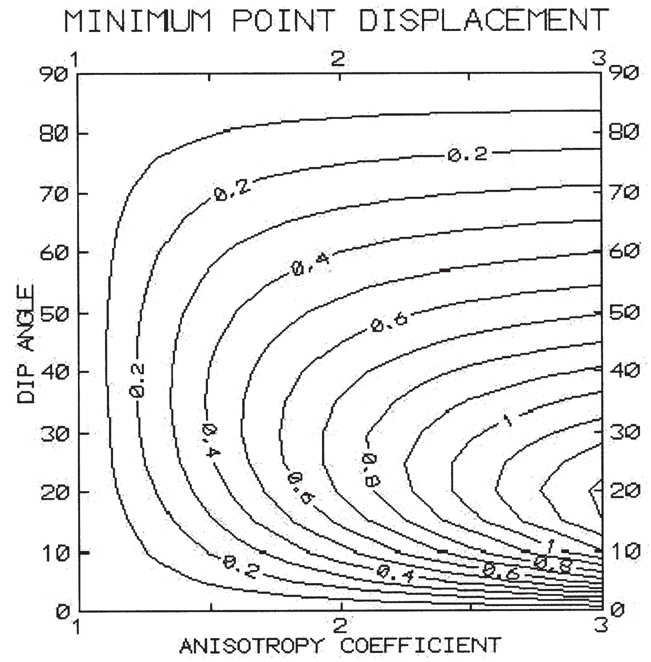

Fig. 2. Displacement x_0/h against θ and A (SKIANIS & HERNÀNDEZ 1999).

Using relations (2) and (3), the displacement of the SP anomaly x0 and the error Δh in depth calculation were defined (SKIANIS & HERNÀNDEZ, 1999). In (Fig. 2) and (Fig. 3) it is shown how the anisotropy parameters influence x_0 and Δh. The displacement x_0 is in the

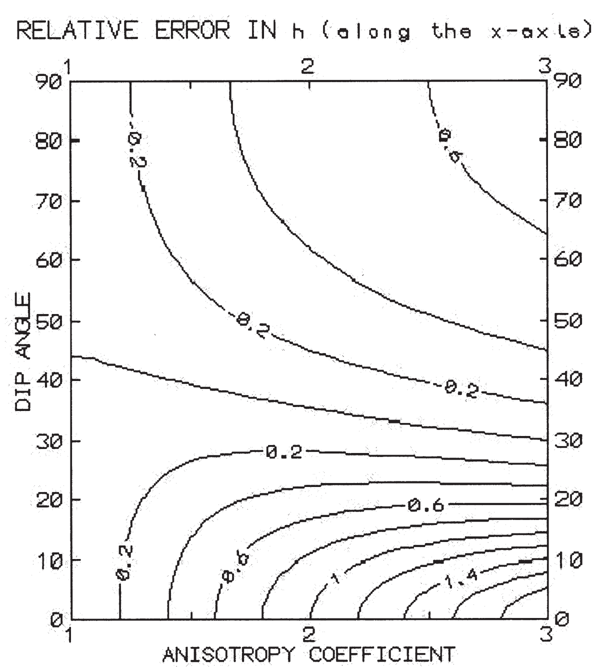

Fig. 3. Relative error Δh/h against θ and A (SKIANIS & HERNANDEZ, 1999).

opposite direction of the horizontal side of the schistosity angle. The displacement is especially big when the schistosity angle is about 20º. The error in depth calculation is negative (apparent depth less than real depth) when the schistosity angle is more than 30º- 40º. When the schistosity angle is less than 30º- 40º, the error is positive (apparent depth less than real depth).

It is important to mention that at the surface of a homogeneous and isotropic ground, the equipotential contours of the self potential field of a point pole are homocentric circles. On the other hand, at a homogeneous and transversely anisotropic ground the equipotential contours are ellipses. The ellipticity gets stronger when the anisotropy coefficient A is increased. When the level of schistosity is horizontal (schistosity angle equal to zero), the equipotential contours are circular, for any value of A. When the schistosity angle is increased, the ellipticity gets stronger.

THE MODEL OF THE VERTICAL DIPOLE

The model of the vertical dipole is presented in (Fig. 4). The expression for the sp anomaly which is produced by a vertical electric dipole with current intensity –I at the upper pole and +I at the lower pole, length L, upper pole depth h and lower pole depth H, may be easily deduced by relation (1). The self potential anomaly V(x) is given by:

$$V(x) = -\frac{I'\rho_m}{2\pi}\cdot\left[\frac{1}{\sqrt{(x-x_{01})^2+h'^2}}-\frac{1}{\sqrt{(x-x_{02})^2+H'^2}}\right] \quad (4)$$

x_{01}, x_{02}, h', H' and I' are defined by the relations:

$$x_{01} = \frac{(A^2-1)\sin\theta\cdot\cos\theta}{\cos^2\theta + A^2\sin^2\theta}\cdot h \quad (5)$$

$$x_{02} = \frac{(A^2-1)\sin\theta\cdot\cos\theta}{\cos^2\theta + A^2\sin^2\theta}\cdot (h+L) \quad (6)$$

$$h' = \frac{Ah}{\cos^2\theta + A^2\sin^2\theta} \quad (7)$$

$$H' = \frac{A(h+L)}{\cos^2\theta + A^2\sin^2\theta} \quad (8)$$

$$I' = \frac{I}{\sqrt{\cos^2\theta + A^2\sin^2\theta}} \quad (9)$$

respectively.

The quantities x_{01}, h', x_{02} and H' show that the SP anomaly at a homogeneous and transversely anisotropic ground, behaves as if it were produced by an inclined dipole in a homogeneous and isotropic ground with a negative pole at location (x_{01}, h') and a positive pole at (x_{02}, H') (see Fig. 4). Although the dipole is vertical, the SP anomaly at ground surface has a negative and a positive centre, because of anisotropy.

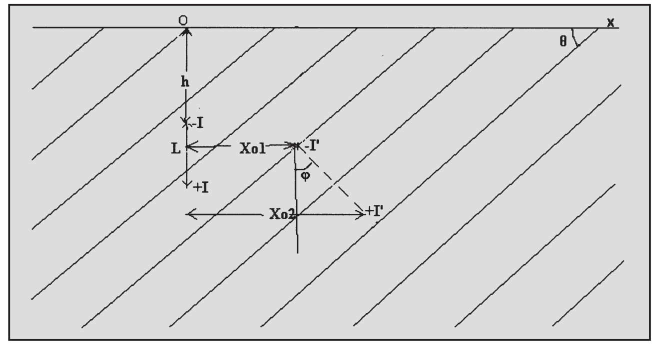

Fig. 4. The model of the vertical dipole with a finite length at a homogeneous and transversely anisotropic ground. The vertical dipole is represented with the solid line –I +I. The "equivalent" inclined dipole at an assumed isotropic medium is represented with the dashed line –I' +I'.

The study of the self potential field of a point pole and a vertical dipole at a homogeneous and transversely anisotropic ground shows that anisotropy deforms the SP anomaly. These deformations may produce significant errors in the quantitative interpretation of the parameters of the polarized body. New quantitative interpretation techniques of self potential anomalies have to be developed in such a way that ground anisotropy is taken into account.

QUANTITATIVE INTERPRETATION OF SELF POTENTIAL ANOMALIES IN A TRANSVERSELY ANISOTROPIC GROUND

Two models which are frequently used in quantitative interpretation of SP anomalies are the polarized sphere and the inclined sheet.

The model of the polarized sphere of depth h, polarization angle α and radius l (l << h) is presented in Fig. 5. The sphere is embedded in a homogeneous and transversely anisotropic ground.

The expression for the SP anomaly V(x) is (Skianis *et al*., 2000):

$$V(x) = K\frac{x + Sh/U + h'\cot\omega}{[(x+Sh/U)^2 + h'^2]^{3/2}} \quad (10)$$

K is a constant which depends on the polarization of the sphere and the anisotropy parameters. The exact mathematical expression for K may be found in Skianis *et. al*. (2000).

h' is the apparent depth and it is defined by relation (7).

The quantities U, S and ω are defined by:

$$U = \cos^2\theta + A^2\sin^2\theta \quad (11)$$

$$S = (1 - A^2)\sin\theta.\cos\theta \quad (12)$$

$$\cot\omega = \frac{A\cot\alpha}{\cos^2\theta + A^2\sin^2\theta + (1-A^2)\sin\theta.\cos\theta.\cot\alpha} \quad (13)$$

respectively

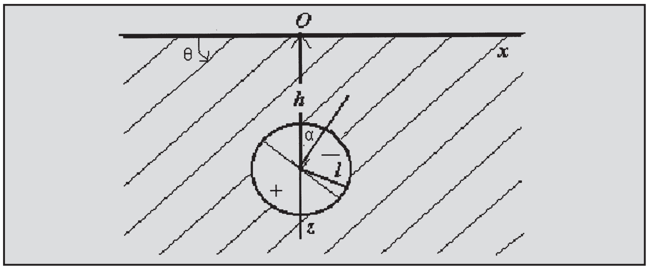

Fig. 5. The model of the polarized sphere in a homogeneous and transversely anisotropic ground.

Taking into account the relations (10), (11), (12) and (13), the following quantitative interpretation method is proposed (SKIANIS *et al.*, 2000):

1. Calculation of the apparent depth h' and the apparent polarization angle ω, assuming a homogeneous and isotropic ground and using any method of quantitative interpretation which has been referred in the bibliography.
2. Calculation of the real depth h and polarization angle α, according to the relations:

$$h = h'\frac{\cos^2\theta + A^2\sin^2\theta}{A} \qquad (14)$$

$$\alpha = \arctan\frac{A + \cot\omega.(A^2 - 1)\sin\theta.\cos\theta}{\cot\omega.(\cos^2\theta + A^2\sin^2\theta)} \qquad (15)$$

This quantitative interpretation method may work if the anisotropy parameters are a priori known. This is possible if dc current geophysical prospecting and geological observations are carried out.

It is also possible to make a quantitative interpretation of an SP anomaly which is produced by an inclined sheet in a homogeneous and transversely anisotropic ground with similar methodology (first assume an isotropic ground and then calculate the real parameters of the inclined sheet). The method is extensively presented in SKIANIS *et al.* (2001).

THE TRANSVERSE ANISOTROPIC GROUND ACTS LIKE A FILTER WHICH MODULATES THE SELF-POTENTIAL ANOMALY

So far we have studied the behavioor of the SP anomaly in the space domain. Using the Fourier transform, it is possible to describe the self potential anomaly at spatial frequency (wave number) domain.

The Fourier transform U(u) of the potential anomaly V(x) is:

$$U(u) = \int_{-\infty}^{\infty} V(x) \cdot e^{-iux} dx \qquad (16)$$

u is the spatial frequency.

The self potential anomaly $V_{iso}(x)$, which is produced by a point pole in a homogeneous and isotropic ground with resistivity ϱ_m, is given by:

$$V_{iso}(x) = \frac{I\rho_m}{2\pi\sqrt{x^2 + h^2}} \qquad (17)$$

The expression for the SP anomaly $V_{an}(x)$, which is produced by the same point pole in a homogeneous and transversely anisotropic ground, is given by relation (1), for y=0. x is the horizontal axis which is perpendicular to the strike of schistosity. $V_{an}(x)$ is given by:

$$V_{an}(x) = \frac{I\rho_m}{2\pi\sqrt{\cos^2\theta + A^2\sin^2\theta} \cdot \sqrt{(x - x_0)^2 + h'^2}} \qquad (18)$$

The effective resistivity ϱ_m of the anisotropic ground is supposed to be equal to the (true) resistivity of the isotropic ground.
Taking into account the relations (16), (17) and (18), and using known Fourier integrals and properties of the Fourier transform (GRADSHTEYN & RYZHIK, 1980), it can be proved that the Fourier transforms $U_{iso}(u)$ and $U_{an}(u)$ of $V_{iso}(x)$ and $V_{an}(x)$, respectively, are given by:

$$U_{iso}(u) = (I\rho_m/2\pi).K_0(hu) \qquad (19)$$

$$U_{an}(u) = \frac{I\rho_m}{2\pi\sqrt{\cos^2\theta + A^2\sin^2\theta}} \cdot K_0(h'u).\exp(-ix_0u) \qquad (20)$$

K_0 is the modified Bessel function of the second kind and zero order.
Combining (19) and (20) gives:

$$U_{an}(u) = U_{iso}(u) \cdot \frac{1}{\sqrt{\cos^2\theta + A^2\sin^2\theta}} \cdot \frac{K_0(h'u)}{K_0(hu)} \cdot e^{-ix_0u} \qquad (21)$$

This means that the transversely anisotropic ground operates as a filter which modulates the SP anomaly of the point pole. In spatial frequency domain, the response F(u) of this filter is given by:

$$F(u) = \frac{1}{\sqrt{\cos^2\theta + A^2\sin^2\theta}} \cdot \frac{K_0(h'u)}{K_0(hu)} \cdot e^{-ix_0u} \qquad (22)$$

The coefficient $(\cos^2\theta + A^2\sin^2\theta)^{-1/2}$ is independent of u, therefore it does not influence the shape of the spectrum of the SP anomaly.

The exponential factor $\exp(-ix_0u)$ produces the shift (displacement) of the SP anomaly in the space domain. x_0u is the phase of the spectral component with spatial frequency u.

The behavior of the ratio $K_0(h'u)/K_0(hu)$ is of specific importance, because it depends on u and determines the amplitude spectrum of the self potential field,.

In Fig. 6 we present the variation of $K_0(h'u)/K_0(hu)$

against u. It can be seen that for h'<h, high spatial frequencies are amplified. On the contrary, when h'>h high spatial frequencies are attenuated. According to relation (3) and Fig. 3, the apparent depth h' is less than the real depth h when the schistosity angle θ is more than 30º- 40º. On the other hand, when θ is less than 30º- 40º, h' is more than h. In fact, the schistosity angle controls the spectral content of the SP anomaly. For a small schistosity angle, the amplitudes of the high spatial frequencies attenuate and the measured SP anomaly at space domain is wide and smooth. On the contrary, for a large schistosity angle the measured SP anomaly at space domain is relatively narrow and presents rapid changes.

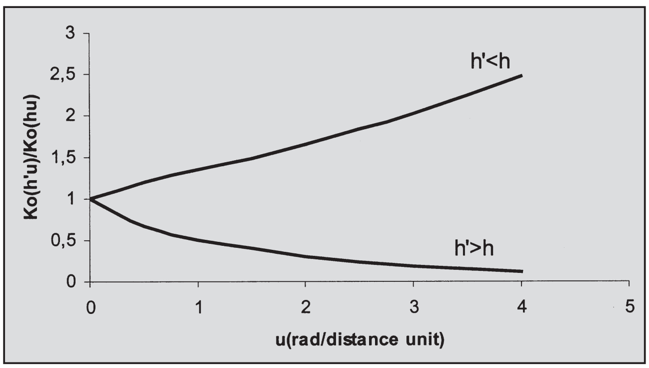

Fig. 6. Variation of the ratio $K_0(h'u)/K_0(hu)$ against u.

Spatial frequency analysis has been extensively used in the quantitative interpretation of SP anomalies, considering an isotropic medium. Relation (21) may be useful in the quantitative interpretation of SP anomalies at spatial frequency domain, taking into account the anisotropy of the ground. The development and application of quantitative interpretation techniques based on this relation, in order to detect ore bodies or subsurface flows of water or heat, may be the subject of future papers.

CONCLUSIONS

The study of the self potential field in a transversely anisotropic ground showed that anisotropy may seriously deform the SP anomaly measured on the ground surface. In the spatial frequency domain, transverse anisotropy acts like a filter which modulates the amplitude and phase of each spectral component. The filter response is mainly determined by the schistosity angle of the ground.

Significant errors may be introduced, if one attempts to calculate the parameters of the polarized body without taking into account the ground anisotropy effect. However, it is possible to make a reliable quantitative interpretation if the anisotropy coefficient and the schistosity angle are known a priori, by conducting dc geoelectrical measurements combined with geological observations.

SP data are usually interpreted assuming certain polarization geometries for the self potential sources. PATELLA (1997) has developed the promising methodology of self potential tomography, which does not assume any a priori knowledge of the shape of the SP source. Self potential tomography works with the assumption of an electrically isotropic ground, which may be homogeneous or non homogeneous. It is interesting to expand this methodology in cases of ground anisotropy, where the SP field of the point pole is described by relations (1) and (21).

The results and conclusions of this paper can be useful in mineral exploration, as well as in hydrogeology, geotechnical applications and geothermal research in order to detect subsurface water flow and leakages or heat flow sources respectively.

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